Determination and use of the soil equivalent in fertilizer experiments

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Abstract
The article presents the bases of a quantitative theory of the response of agricultural crops to mineral fertilization. Several additive and multiplicative empirical mathematical models of yield are described. The main equation represents yield as an intrinsically non-linear function of macro element fertilization. A new quantity is considered - soil equivalent, with the help of which is possible to estimate the quantity of the nutrient in the soil which is readily available to plants in a form equivalent to the applied fertilizers. Methods for determining the value of the soil equivalent and its use in estimating the parameters of mathematical models, as well as the values of fertilizers at which the maximum yield is obtained, are presented. The application of the models and the use of soil equivalent is illustrated by two examples. The first is from a field experiment with wheat at the experimental station in Nikolaevo, Pleven district, Bulgaria. The second is from the "3414" application design of NPK fertilizer for 3-yr-old *Panax notoginseng* in Yunnan Province, China.

Key words: crop response**;** fertilizer experiments; soil equivalent

INTRODUCTION

The dependence between yield and the quantities of nutrients in the soil (macro elements: N, P, K, Ca, Si, Mg, S, and Na; microelements: B, Mo, Zn, Fe, Mn, Cu, Co, Cl) is established through the construction of different mathematical models that describe sufficiently well the experimental data obtained in field and glasshouse (pot) trials. These mathematical models are often called production functions or response functions (Dillon, 1968). One of the problems in the application of the production function yield-fertilization in agrochemical research is the adjustment of the results of field experiments to take into account soil fertility. Throughout the text the output variable or yield is designated by Y, while the input variable or nutrient is designated by X. The response function can be presented with the equation

$Y = f(X).$

The construction of a mathematical model of yield involves three main tasks:

- to select an appropriate model type (analytical expression);
- to determine the constant values (unknown parameters) of the model;
- to study the properties of the resulting model and to determine the optimal values of the input variables.

Several empirical mathematical models of yield have been described in the literature. All of them are simply tools for interpolation in a particular narrow interval and cannot fully describe the phenomenon, because they lack a sound theoretical basis.

A quantitative theory about the response of agricultural crops to mineral fertilization was described (Sadovski, 1984), based on the assumption that the quantity of a given nutrient in the soil can be considered a stimulus for the sowing, while yield can be considered a reaction of the sowing to this stimulus.

The connection between stimulus and reaction has been thoroughly investigated and usually assumes the shape presented in Figure 1 (Ackoff, 1982). Applied to the theory of response, it means the following: The presence of negligible quantities of the nutrient has almost no effect on yield, but after reaching a threshold level the effect begins to increase. The yield increases to a saturation point and starts to decrease at higher concentrations of the nutrient.

The idea that optimal fertilization rates can be determined based on an assessment of the nutrient content of the soil using production functions was put forward by Anderson (1956). Derzhavin & Rubanov (1975) introduced the term soil equivalent for the quantity of the nutrient in the soil which is readily available to plants in a form, equivalent to the applied fertilizers. The soil equivalent can serve as an estimate of the soil fertility in the active ingredient of the applicable mineral fertilizers. Moreover, this assessment will be objective in the sense that it is obtained from the results of field experiments, which are a reflection of the existing natural conditions in this area (Rubanov, 1978). Field experiments show that the soil equivalent has a close correlation with the yield from the control, i.e. without fertilizer application (Derzhavin & Rubanov, 1980). The present article aims to show the place of the soil equivalent in the mathematical model of yield and give a method for its determination and use.

MATERIALS AND METHODS

Yield is essentially a stochastic (random) variable, while nutrient quantity can be either an independent (determined) variable or a random variable. The above considerations lead to the following principle assumptions of the quantitative theory about the response of agricultural crops:

1) The response function $Y = f(X)$, depending on the content of a particular nutrient in the soil X, is a continuous function monotonically increasing in a finite interval $[0, X_{max}]$, and then monotonically decreasing to zero.

2) In the absence of a given nutrient, the yield is equal to zero ($f(0) = 0$). Therefore, the function is non-negative in the indicated interval ($f(X) = 0$ when $X = 0$).

The above assumptions lead to the conclusion that the function f(X) has a continuous first derivative, its second derivative is negative and a maximum positive yield value exists.

$Y_{max} = f(X_{max}) > 0.$

The above requirements are only met by additive models without constant terms, for example

$Y = bX + cX^2$,

$Y = b\sqrt{X} + cX$

or multiplicative models, such as

$Y = aX^b e^{cX}$,

$Y = aX^b(k - X)^c$

A significant error in a many of the known models is that the argument X on their right-hand side represents the introduced quantity of mineral fertilizer, without taking into account the quantity of nutrients already available in the soil.

This theory expresses the quantity of a given nutrient X as a sum of the initial level of the nutrient X_0 and the quantity introduced with fertilizers F

$$X = X_0 + F. \quad (1)$$

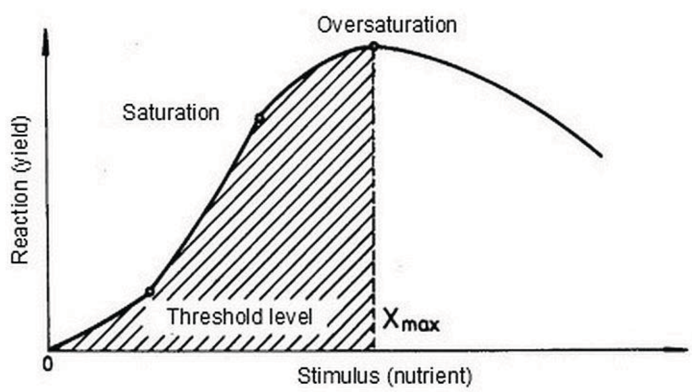

Figure 1. The connection "stimulus-reaction"

The quantity X_0 of the nutrient in the soil which is readily available to plants is the soil equivalent (Derzhavin & Rubanov, 1975). Knowing the required amount of nutrient substance to obtain a given yield and a soil equivalent, one can find the required amount of fertilizer to obtain a given yield. It will be determined as the difference between the required amount of nutrient substance and the soil equivalent (Rubanov, 1978).

The main equation represents yield as an intrinsically non-linear function of macro element fertilization and has the expression

$$Y = f(X) = a(X_0 + F)^b \exp[c(X_0 + F)]. \quad (2)$$

This multiplicative equation is a composition of a power function and an exponential function. Thus, it contains the mechanism of homeostatic feedback.

The crop response function defined above allows the following theoretical conclusions: The function is defined for every non-negative X. When $a > 0$ and $X > 0$ it has only positive values. At $b > 0$ the curve passes through the origin and at $c<0$ the curve asymptotically approaches zero. Therefore, the conditions $a > 0$, $b > 0$, and $c < 0$ are necessary and sufficient to satisfy the two main requirements of the theory. The properties of the response function can easily be derived using the first and the second derivative. The first derivative is

$$\frac{dY}{dX} = \frac{Y}{X}(b + cX) \quad (3)$$

When the first derivative is equal to zero and the resulting equation is solved for X we obtain the input that gives a maximum output

$$Y_{max} = -b/c, \text{ therefore } F_{max} = -b/c - X_0. \quad (4)$$

The second derivative is

$$\frac{d^2Y}{dX^2} = \frac{Y}{X^2}\; b(b-1) + 2bcX + c^2X^2 \quad (5)$$

When the right-hand side of the equation is equal to zero, we can determine the respective inflection points

$$X_{1,2} = \frac{-b \pm \sqrt{b}}{c} \text{ when } F_{1,2} = \frac{-b \pm \sqrt{b}}{c} - X_0. \quad (6)$$

The best conditions for functioning in crop response are determined by the system of input values that optimizes a particular target function.

The main difficulty in using the crop response function defined above is the determination of the parameters of the non-linear function. They can be calculated with computer technology by using the non-linear least-squares method (Demidovich et al., 1967; Dixon, 1972). If the constant X_0 is known, then the other three parameters can easily be determined from the main equation (2) by applying a logarithm

$$\ln Y = \ln a + b \ln(X_0 + F) + c(X_0 + F). \quad (7)$$

An estimation of the soil equivalent X_0 is possible when the response function is approximated by a second-degree polynomial (Rubanov, 1978).

$$Y = a_0 + a_1F + a_2F^2. \quad (8)$$

Then X_0 is the absolute value of the negative root of the quadratic equation with coefficients a_0, a_1, and a_2 (See Figure 2).

$$X_0 = \left| \frac{-a_1 + \sqrt{a_1^2 - 4a_0a_2}}{2a_2} \right|. \quad (9)$$

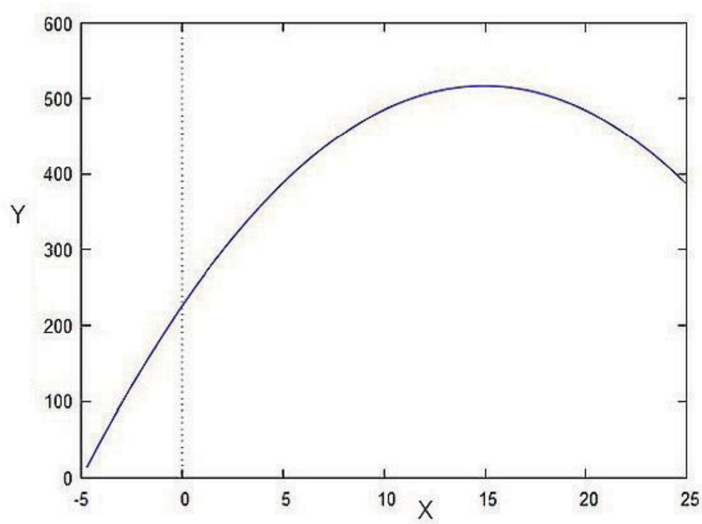

Figure 2. Response function and soil equivalent

In the case of two nutrients, the main equation acquires the form

$$Y = f(X_1, X_2) = a(X_{01} + F_1)^{b_1}(X_{02} + F_2)^{b_2}$$
$$\exp[c_1(X_{01} + F_1) + c_2(X_{02} + F_2)]. \quad (10)$$

Here F_1 and F_2 are the quantities of the two microelements introduced with the fertilizer, while X_{01} and X_{02} are their respective soil equivalents.

The point of biological maximum, i.e. the values of F_1 and F_2 for which the response function reaches its maximum, can be determined by solving the system

$$\left\| \begin{array}{l} \dfrac{\delta Y}{\delta X_1} = 0 \\ \dfrac{\delta Y}{\delta X_2} = 0 \end{array} \right. . \quad (11)$$

More explicitly, this is a system of equations in two unknowns

$$\left\| \begin{array}{l} \dfrac{Y}{X_1}(b_1 + c_1 X_1) = 0 \\ \dfrac{Y}{X_2}(b_2 + c_2 X_2) = 0 \end{array} \right. \quad (12)$$

that gives the following solution for the maximum

$$F_{1\max} = -\frac{b_1}{c_1} - X_{01}, \quad F_{2\max} = -\frac{b_2}{c_2} - X_{02}. \quad (13)$$

Since the response function with equation (10) is non-linear concerning the unknown parameters *a*, b_1, b_2, c_1, c_2, X_{01} and X_{02}, their determination requires the use of the non-linear least-squares method. By analogy to the case with one variable, if the value of the soil equivalents X_{01} and X_{02} and the two nutrients is known, then the logarithmic transformation of the equation gives a function which is linear concerning the other parameters.

$$\ln Y = \ln a + b_1 \ln(X_{01} + F_1) + b_2 \ln(X_{02} + F_2) -$$
$$c_1(X_{01} + F_1) + c_2(X_{02} + F_2). \quad (14)$$

The approximate values of X_{01} and X_{02} can be determined through an approximation of the response function by a second-degree polynomial which includes the interaction between the nutrients

$$Y = a_0 + a_1 F_1 + a_2 F_2 + a_3 F_1^2 + a_4 F_2^2 + a_5 F_1 F_2. \quad (15)$$

When $F_2 = 0$ the equation is $a_0 + a_1 F_1 + a_3 F_1^2 = 0$, and its negative root is

$$X_{01} = \left| \frac{-a_1 + \sqrt{a_1^2 - 4a_0 a_3}}{2a_3} \right|. \quad (16)$$

By analogy, when $F_1 = 0$ the equation is $a_0 + a_2 F_2 + a_4 F_2^2 = 0$, and the value is

$$X_{02} = \left| \frac{-a_2 + \sqrt{a_2^2 - 4a_0 a_4}}{2a_4} \right|. \quad (17)$$

A further generalization of the response function for k nutrients introduced with mineral fertilization gives the equation of yield

$$Y = f(X_1, X_2, ..., X_k) =$$
$$a \prod_{i=1}^{k} (X_{0i} + F_i)^{b_i} \exp[\sum_{i=1}^{k} c_i (X_{0i} + F_i)]. \quad (18)$$

The principal assumptions of the new theory on crop response to mineral fertilization are sufficiently generalized and any further attempts for its development may only relate to the analytical expression of its main equation (2). For example, an alternative form of this equation representing yield as a non-linear function of macro element fertilization is

$$Y = g(X) = a(X_0 + F)^b [k - (X_0 + F)]^c. \quad (19)$$

This also is a multiplicative equation employing the mechanism of negative feedback. It has 5 unknown parameters – X_0, *a*, *b*, *c*, and *k*. X_0 is the soil equivalent. Here *k* is interpreted as the quantity of nutrient X that causes depression and decreases the yield to zero. The equation can be used to reach theoretical conclusions and make calculations by analogy to the approach described above.

Again the logarithmic transformation of the equation gives a function which is linear concerning the other parameters

$$\ln Y = \ln a + b\ln(X_0 + F) + c\ln[k - (X_0 + F)]. \quad (20)$$

Future research could assess the practical applicability of this model in processing data from field trials.

RESULTS

According to Wimble (1980), good response models must meet two main requirements:

1) To determine the slope of the increasing part of the curve and the slope after the maximum;

2) To determine accurately the yield when fertilization is close to optimal and the necessary fertilizer application.

The analytical conclusions following from the main yield equation (2) in the one-dimensional, (10) two-dimensional and (18) k-dimensional case fulfill these two criteria. The theory of crop response to mineral fertilization sets some requirements for the planning of experiments and the statistical evaluation of the function's parameters. It provides the opportunity to explain the variation in response in time and space depending on the initial soil fertility and to predict the quantity of attainable yield depending on the applied mineral fertilizers.

Example 1. Finding soil equivalent from a field experiment with wheat.

An excerpt from the data of yield at the experimental station Nikolaevo, Pleven district was used (Yearbook, 1971). The experiment was performed according to the scheme 4x4x3 (nitrogen 0,12,18,24; phosphorus 0,8,16,24; potassium 0,12,24 kg/da). Data on phosphorus variation were calculated at a fixed level of nitrogen (N = 16 kg/da) and potassium (K = 0 kg/da). Corresponding regression equation is

$Y = 226.607 + 38.854*F -1.2984*F^2$.

According to the formula (9), a value of the soil equivalent $X_0 = -4.99$ is obtained, which means that 5 kg/da phosphorus is readily available to plants in a form, equivalent to the applied fertilizers. From here the maximum yield is obtained at the following fertilizer values:

$Y_{max} = - b/c = -38.854/-1.2984 = 29.92$ kg/da, therefore $F_{max} = -b/c - X_0 = 29.92 - 4.99 = 24.93$ kg/da.

Using the obtained value of X_0 with the help of equation (7) we can find the constants a, b, c of the transcendental function with the form

$Y = 164.9 X^{0.5733}\, e^{-0.0316 X}$.

Example 2. Data are taken from the "3414" application design of NPK fertilizer for 3-yr-old *Panax notoginseng* in Yunnan Province, China (Xia et al., 2016). A one-factor quadratic regression model was constructed for P_2O_5 fertilizer, when fixing the application level of N and K fertilizers at 22.5 kg/667 m^2 and 45 kg/667 m^2, respectively

$Y = 1501.29 + 9.07*X_2 - 0.208*X_2^2$

where Y represents the dried root weight of 100 plants (g) as a dependent variable and X_2 is phosphorus fertilizer.

When applying formula (9) soil equivalent of phosphorus was calculated with a value of $X_0 = -65.907$ and its absolute value is 65.91 kg/667 m^2.

Analogously to Example 1, the corresponding transcendental function can be determined and its properties - maximum and inflection points - can be studied.

CONCLUSIONS

The bases of a quantitative theory of the response of agricultural crops to mineral fertilization are described. The article presents several additive and multiplicative empirical mathematical models of yield as a dependent variable from applied fertilizers. The new quantity - soil equivalent, is discussed with the help of which is possible to estimate the quantity of the nutrient in the soil which is readily available to plants. The presented examples from experiments conducted in Bulgaria and China confirm the theoretical meaning and practical benefit of using the soil equivalent in processing the results of field experiments.

The proposed quantity soil equivalent allows solving several problems of agrochemical service of agriculture and reducing the cost of additional field experiments with fertilizers to assess their effectiveness. Another perspective for research is to establish the correlation between calculated values of the soil equivalent and results from laboratory analyses of the same soils regarding the content of nutrients available to plants. Such studies would allow the direct application of the quantitative theory in an automated system for soil and agrochemical service in agriculture.

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