Optimization of maize yield by fuzzy regression. II

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Abstract
The purpose of the present study is to determine the influence of the main nutrients nitrogen, phosphorus, potassium, and silicon with the help of the theory of fuzzy sets in the conditions of a vegetation (pot) experiment with maize on soil from experimental fields Bozhurishte (Pellic Vertisol) and Tslapitsa (Eutric Fluvisol). The design of multifactorial experiments allows the assessment of actions and interactions of four factors, varying on five levels. The use of fuzzy regression techniques is appropriate in pot experiments when the results obtained are influenced by multiple random factors during the growing season. Statistical analysis of data establishes trends in maize nutrition.

Keywords: pot experiment; maize; yield; fuzzy regression

INTRODUCTION

Experimental research is not limited to a specific area or type of idea. By being able to isolate specific variables, it becomes possible to determine if a potential outcome is viable. This provides a tremendous advantage in the ability to find accurate results (Gaille, 2017). Field experiments are studies using an experimental design that occur in a natural setting. A field experiment is a main and most objective method for studying the theoretical and practical problems of agriculture (Shanin, 1977).

Pot experiments, as a complement to field measurements, allow the investigation of plants under controlled conditions, without distracting the effects of heterogeneous environmental factors (Kawaletz et al., 2014; Epee, 2018). Growing plants for experimental purposes remains an art, requiring in-depth knowledge of physiological responses to the environment. Some of the most overlooked factors in that respect are pot size, and the fact that nutrient and water supply strongly interact with plant size (Poorter et al., 2012). In general, as pot size increases plant leaf area, shoot biomass, and root biomass increase. The growth rates of shoots and roots are interdependent (NeSmith & Duval, 1998).

The optimal values of the main nutrients nitrogen (N), phosphorus (P), potassium (K), and silicon (Si) can be established after performing a regression analysis of the experimental data. The results of the pot trials are influenced by multiple random factors during the growing season. This justifies the use of fuzzy regression methods. Different methods have been applied to find a solution for fuzzy linear systems. A general solution of m×n fuzzy linear systems is given in (Mikaeilvand & Noeiaghdam, 2012), where the original system is replaced by two m×n crisp linear systems.

The theory of fuzzy sets allows us to structure in the best way everything that is not separated by very precise boundaries, for example, the storage of different soils with nutrients. This makes these methods suitable for establishing a tolerance for fertilizing crops with different substances.

The purpose of the present study is to determine the influence of the main nutrients nitrogen, phosphorus, potassium, and silicon on the yield of maize with the help of the theory of fuzzy sets in the conditions of a pot experiment on leached smolnitsa (Pelliv Vertisol) and alluvial-meadow soil (Eutric Fluvisol).

MATERIALS AND METHODS

Data from vegetation experiments with maize on leached smolnitsa (Pellic Vertisol) and alluvial-meadow soil (Eutric Fluvisol) (FAO, 2015) derived in 2020 by a team under a project funded by the Scientific Research Fund (Lozanova et al., 2022; Petkova & Sadovski, 2022; Sadovski et al., 2022) are used. Mineral fertilizers - N (ammonium nitrate), P (superphosphate), K (potassium sulfate), and Si (diatomic earth, which represents 89-95% silicon in amorphous form) are applied. The experiment includes 16 variants of fertilization in three replications with experimental pots – 4 kg.

The design of multifactorial experiments allows the assessment of actions and interactions of four factors, varying on five levels. Its configuration consists of 8 star points, one central point, one zero point, and 6 points, which are part of Rechtschafner's saturated design with two levels (Sadovski, 2020). The design of treatments is presented in Table 1.

The indicators that were observed and measured during the growing season were plant height on three dates, plant weight, and root weight at the end of the experiment. Figures 1 and 2 present the variants of the vegetation experiment in Bozhurishte.

Table 1. Pot experiment – quantities of fertilizers (g/pot); values of crisp coefficients matrix A

Variant	N X_1	P X_2	K X_3	Si X_4
1	0	0	0	0
2	0	160	140	800
3	400	160	140	800
4	200	0	140	800
5	200	320	140	800
6	200	160	0	800
7	200	160	280	800
8	200	160	140	0
9	200	160	140	2000
10	200	160	140	800
11	300	240	70	400
12	300	80	210	400
13	300	80	70	1200
14	100	240	210	400
15	100	240	70	1200
16	100	80	210	1200

Figures 3 and 4 show the plant height in successive measurements of the two soils. The coincidence of the results of the group of variants 3, 4, and 5, as well as the group 10, 11, 12, and 13 for both soils can be seen. The maximum is observed for variant 11, which is N = 300, P = 240, K = 70, and Si = 400 g/pot.

The fuzzy set theory provides a strict mathematical framework in which vague conceptual phenomena can be precisely and rigorously studied. Following Zimmermann (1992), a fuzzy number may be defined as *F = (b, g, h);* where *b* denotes the center (or mode), *g* and *h* are the left spread (L) and right spread (R), respectively, L and R denote the left and right shape functions. A popular fuzzy number is the triangular fuzzy number (see Figure 5).

In this paper, the main aim is using of a method where right - hand - side is a fuzzy vector and the coefficients matrix is crisp. Crisp means - something clearly defined, and deterministic. When we have crisp explanatory variables X_i, (i = 1, ..., n) and a fuzzy dependent variable $Y_i \equiv$ (b, g, h), (i = 1, ..., m), a model capable to incorporate the possible influence of the magnitude of the centers on the spreads, can be taken into account (D'Urso, 2003).

From all replications of experimental data X_i, (i = 1, …, 9) are derived the quantities

g_i = Min (X_i); b_i = Average (X_i); h_i = Max (X_i).

There is a simple solution approach to solving a general fuzzy system of linear equations (Mosleh et al., 2011). In the case of a fully fuzzy linear system $A \otimes x = \tilde{b}$ with a new notation *A(A, M, N)*, where *A*, *M*, and *N* are three crisp matrices, of the same size *A*, the matrices *A*, *M*, and *N* are called the center matrix, the left and right spread matrices, respectively.

In our paper, the coefficient matrix is considered as real crisp, whereas the unknown variable vectors are considered fuzzy. In this case, the matrices *M* and *N* are zero matrices. Using matrix notation we have

$$A \otimes x = \tilde{b}$$

or in expanded form

$$\begin{cases} (a_{11} \otimes x_1) \oplus (a_{12} \otimes x_2) \oplus \ldots \oplus (a_{1n} \otimes x_n) = \tilde{b}_1 \\ (a_{21} \otimes x_1) \oplus (a_{22} \otimes x_2) \oplus \ldots \oplus (a_{2n} \otimes x_n) = \tilde{b}_2 \\ . \\ (a_{m1} \otimes x_1) \oplus (a_{m2} \otimes x_2) \oplus \ldots \oplus (a_{mn} \otimes x_n) = \tilde{b}_n \end{cases} \quad (1)$$

Figure 1. Pot experiment – Bozhurishte variants 1 to 8

Figure 2. Pot experiment – Bozhurishte variants 9 to 16

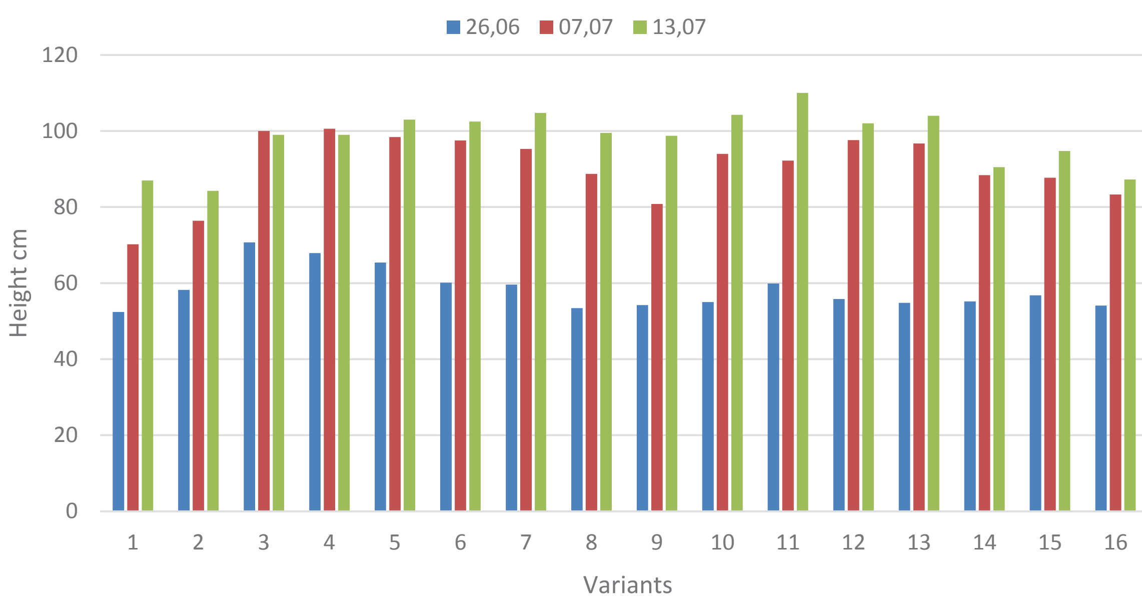

Figure 3. Height of maize on Pellic Vertisol

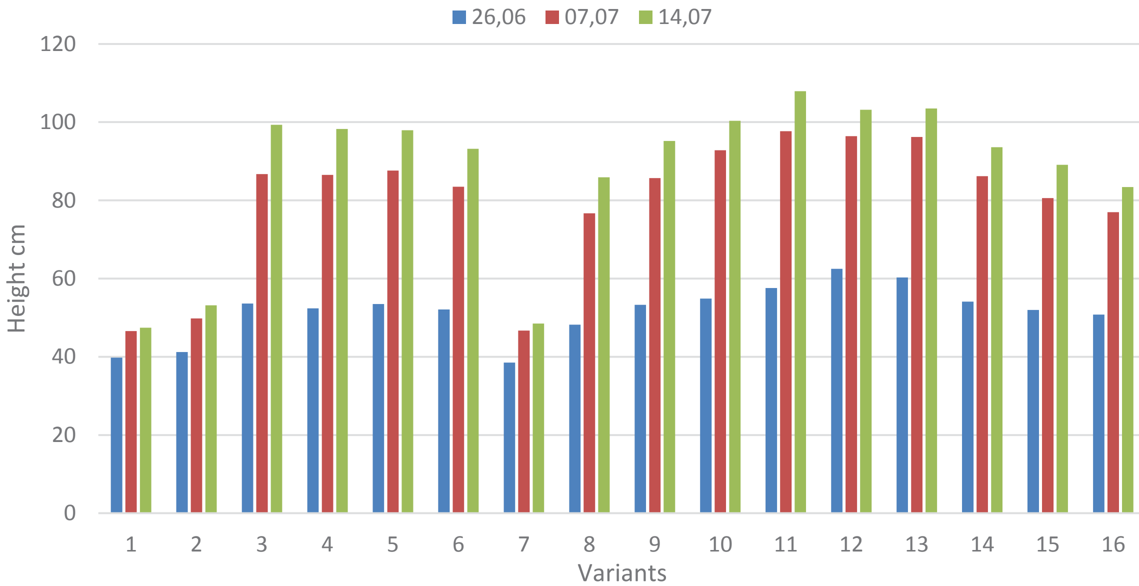

Figure 4. Height of maize on Eutric Fluvisol

where the crisp coefficient matrix is

$A = (a_{ij})$, $(i=1, ..., m; j=1,..., n)$

and $b_i = (b_i, g_i, h_i)$ are nonnegative fuzzy numbers.

For calculate (1) the following simple sequence is used:

1. Singular value decomposition is made

$A = U\Sigma V^t$, (2)

where U and V are orthogonal matrices; and Σ is a diagonal matrix.

2. Pseudo-inverse matrices Σ^+ and $A^+ = V\Sigma^+U^t$ are found.

The following dependencies exist

$$Ax = b, \quad Ay = g, \quad Az = h. \qquad (3)$$

3. From them consecutively the unknown values are calculated

$$x = A^+b, \quad y = A^+g, \quad z = A^+h. \qquad (4)$$

From calculated values of x, y, and z we can find the fuzzy solution

$b = (b, g, h)$,

where $b = Ax$, $g = Ay$, $h = Az$.

Calculations by the method described are done with the free software package GNU Octave, version 6.4.0. (Octave Project, 2021).

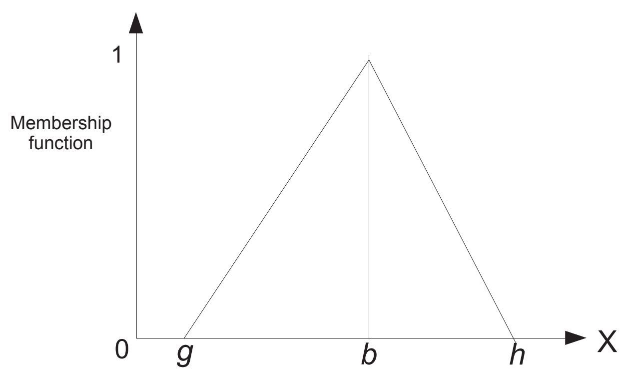

Figure 5. Triangular fuzzy number

Regression analysis of data for the yield of Maize from a field experiment in Bozhurishte and Tsalapitsa is performed using the equation

$Y = b_0 + b_1N + b_2P + b_3K + b_4Si$

In this regression equation, the obtained solution for the fuzzy yield (g, b, h) is considered as the dependent variable Y. Analysis was performed with Prism 9 software (GraphPad, 2020).

RESULTS AND DISCUSSION

The unknown variable vectors we are looking for are:

Yield = (Y_g, Y_b, Y_h),

Explanatory variables for the analysis (see Table 1) are:

X_1 = N (g/pot),
X_2 = P (g/pot),
X_3 = K (g/pot),
X_4 = Si (g/pot).

Fuzzy variable vectors for Bozhurishte – 2020 year, as well as for Tsalapitsa – 2020 year are presented in Tables 2 and 3.

Table 2. Fuzzy variable vectors for Bozhurishte

Variant	Input data			Output results		
	g	b	h	Yg	Yb	Yh
1	10,25	16,18	22,53	0,00	0,00	0,00
2	10,54	15,41	21,63	14,04	15,47	14,54
3	28,43	99,71	191,08	37,16	120,34	229,38
4	19,83	58,19	112,11	17,91	53,83	103,23
5	28,24	73,27	126,64	33,29	81,97	140,69
6	25,84	67,94	123,72	22,65	62,34	111,08
7	31,24	92,75	173,27	28,55	73,47	132,84
8	29,55	77,27	140,23	22,20	72,06	137,03
9	27,14	42,67	56,99	30,70	61,66	99,36
10	25,78	73,04	134,03	25,60	67,90	121,96
11	33,66	114,38	211,85	32,05	100,45	187,13
12	29,32	83,45	160,64	27,31	91,95	179,28
13	28,94	95,05	184,45	27,76	82,22	153,34
14	19,40	38,97	63,18	23,44	53,58	90,58
15	32,88	55,30	82,05	23,89	43,86	64,64
16	21,25	42,47	66,84	19,15	35,35	56,79

Table 3. Fuzzy variable vectors for Tsalapitsa

Variant	Input data			Output results		
	g	b	h	Yg	Yb	Yh
1	0,24	1,72	3,75	0,00	0,00	0,00
2	0,46	2,22	4,81	6,52	8,27	10,42
3	25,30	45,64	64,33	31,10	49,84	69,22
4	25,33	42,23	57,29	15,64	26,44	37,60
5	22,33	33,53	45,65	21,98	31,66	42,04
6	24,91	33,71	42,35	20,79	30,92	41,34
7	0,28	1,67	3,70	16,83	27,19	38,30
8	18,88	27,98	37,50	13,48	21,53	30,10
9	26,16	38,77	52,74	26,81	40,34	54,41
10	24,03	43,10	61,76	18,81	29,05	39,82
11	20,67	31,83	42,57	24,86	37,92	51,53
12	24,49	35,65	50,63	19,72	33,45	47,79
13	23,82	36,93	53,39	27,03	42,83	59,04
14	19,32	27,01	36,21	10,59	15,27	20,61
15	20,32	24,98	30,23	17,90	24,66	31,85
16	17,54	26,61	34,23	12,75	20,18	28,11

Multiple regression analysis of fuzzy solutions for the yield $Y(Y_g, Y_b, Y_h)$ is given in Tables 4 and 5.

Goodness of Fit - R squared = 0.3615.

The positive effect of nitrogen and phosphorus is visible, while the influence of potassium and silicon is insignificant.

Goodness of Fit - R squared = 0.6551.

The positive effect of nitrogen, phosphorus, and silicon is visible, while the influence of potassium is insignificant.

Table 4. Regression analysis of Bozhurishte fuzzy solution

Variable	b	Std.err.	|t|	p
N	0.28570	0.05565	5.1340	<0,0001
P	0.08434	0.06956	1.2120	0.2318
K	0.04617	0.07950	0.5807	0.5644
Si	-0.00660	0.01252	0.5269	0.6009

Table 5. Regression analysis of Tsalapitsa fuzzy solution

Variable	b	Std. err.	|t|	p
N	0.10410	0.01179	8.8310	<0,0001
P	0.01666	0.01474	1.1300	0.2646
K	-0.01278	0.01685	0.7586	0.4522
Si	0.00941	0.00265	3.5470	0.0009

CONCLUSIONS

The influence of the main nutrients nitrogen (N), phosphorus (P), potassium (K), and silicon (Si) on the yield of Maize was established after performing a regression analysis of the pot experimental data. The interpretation of the obtained results shows the need for higher doses of Nitrogen and Phosphorus for the Pellic Vertisol. Potassium and Silicon do not have any significant effect. Maize in Tsalapitsa requires Nitrogen and Phosphorus and a larger amount of Silicon as stock fertilization. Potassium does not have any significant effect.

In this article, we show the efficiency of the proposed method for solving linear fuzzy regression. This scheme for finding the positive solution of the fuzzy systems, when parameters are positive, it turns out quite satisfactory. It can be concluded that the fuzzy set techniques are promising for future research in other agricultural crops as well.

Disclaimer

The Author does not imply the expression of any opinion about the software mentioned in the article.

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